Ghostly action at a distance: a non-technical explanation of the Bell inequality

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We give a simple non-mathematical explanation of Bell's inequality. Using the inequality, we show how the results of Einstein-Podolsky-Rosen (EPR) experiments violate the principle of strong locality, also known as local causality. This indicates, given some reasonable-sounding assumptions, that *some* sort of faster-than-light influence is present in nature. We discuss the implications, emphasizing the relationship between EPR and the Principle of Relativity, the distinction between causal influences and signals, and the tension between EPR and determinism.

I. INTRODUCTION

The recent announcement of a "loophole-free" observation of violation of the Bell inequality [1] has brought renewed attention to the Einstein-Podolsky-Rosen (EPR) family of experiments in which such violation is observed. The violation of the Bell inequality is often described as falsifying the combination of "locality" and "realism". However, we will follow the approach of other authors including Bell [2–4] who emphasize that the EPR results violate a single principle, strong locality.

Strong locality, also known as "local causality", states that the probability of an event depends only on things in the event's past light cone. Once those have been taken into account the event's probability is not affected by additional information about things that happened outside its past light cone.

Given some reasonable-sounding assumptions about causation (see Sec. III), the violation of strong locality in EPR experiments implies that there are causal influences that travel faster than light. The main goal of this paper is to give an extremely simple non-technical explanation of how EPR experiments lead to this striking conclusion. We do this by mapping the experiment onto a situation where twins are separated and then asked questions to test whether they can influence each other via faster-than-light telepathy.

Since the EPR results force us to accept that nature does not respect strong locality, it is natural to ask how the results cohere with other locality principles. Is there a sense in which the results violate "local realism"? We discuss Einstein's Principle of Relativity (Lorentz invariance) and the principle that signals cannot travel faster than light ("signal locality"). We describe how signal locality arises from the Principle of Relativity, and show how it can be reconciled with EPR results, but only if we accept that nature has an ungraspable aspect, such as indeterminism or some other form of uncontrollability, that prevents the violation of strong locality from leading to faster-than-light signalling.

II. OVERVIEW

To make it clear how EPR experiments falsify the principle of strong locality, we now give an overview of the logical context (Fig. 1). For pedagogical purposes it is natural to present the analysis of the experimental results in two stages, which we will call the "EPR analysis", and the "Bell analysis" although historically they were not presented in exactly this form [4, 5]; indeed, both stages are combined in Bell's 1976 paper [2].

We will concentrate on Bohm's variant of EPR, the "EPRB" experiment. This involves pairs of particles, typically a pair of photons in a spin singlet state. The question at hand is: what general types of theories can account for the observed behavior of these particles? Can strongly local theories do the job? Fig. 1 shows the space of theories of such particles. The inner (red) rectangle encloses the set of strongly local theories, the ones in which the probability of an event depends only on occurrences in the event's past light cone. The upper (green) rectangle encloses the set of theories that are "deterministic",

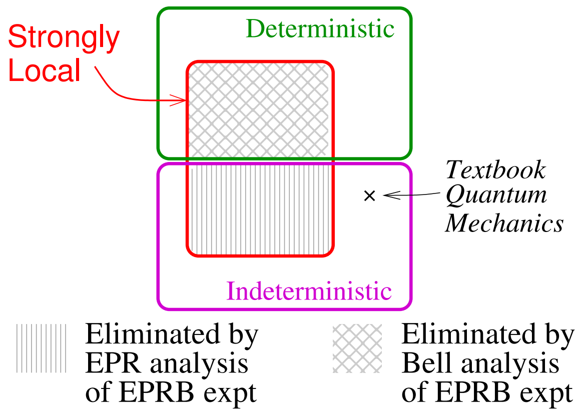

FIG. 1: Venn diagram of the space of theories and the constraints from EPRB experiments. The inner (red) rectangle encloses the set of strongly local theories. The EPR analysis concludes that strongly local theories must be deterministic; the Bell analysis concludes that strongly local theories cannot be deterministic. In combination, these analyses rule out strongly local theories.

meaning that the behavior of the particles is is fully determined in advance without any randomness in the laws of physics. When combined [2], the EPR analysis and the Bell analysis show that no strongly local theory, whether deterministic or indeterministic, can explain the results of EPRB experiments. We now give a brief outline of those analyses, to be expanded in later sections.

The EPR analysis
The EPR analysis [6] starts with the experimental observation that both photons in the EPRB setup show the same behavior when subjected to the same measurement, no matter how far apart they are. The EPR analysis then points out that if strong locality is true then this cannot be due to one photon influencing the other, so they must have been pre-programmed to agree, which requires that the photons have a deterministically-evolving internal state that determines their behavior. In other words, strong locality *requires* determinism to explain the EPRB results. The EPR analysis therefore rules out strongly local theories that are indeterministic (vertical shading in Fig. 1). This sounds like a refutation of quantum mechanics, which is famously an indeterministic theory. However, "textbook" quantum mechanics, as taught in conventional physics courses, explicitly violates strong locality because measurement induces instantaneous collapse of the wavefunction over all of space, so EPR's analysis does not apply directly to textbook quantum mechanics. Rather, it shows that any alternate theory that was strongly local would have to be deterministic. In such a theory the result of measuring a photon's spin would not be random; it would be determined by the state of non-quantum-mechanical "hidden variables" that predetermine the behavior of the photon.

The Bell analysis and the Bell inequality
The second stage of analysis of the EPRB experimental data, which we call the "Bell analysis", destroys the dream of finding a strongly local and deterministic theory to replace quantum mechanics. Bell pointed out that if nature is described by a strongly local and deterministic theory then the behavior of the photon pairs has to obey a constraint called the "Bell inequality" [2, 7]. In Secs. IV and V we will give an elementary explanation of the Bell inequality in terms of testing twins for faster-than-light telepathy. We will show that it arises from the fact that if someone has planned yes-or-no answers to three questions then on two randomly chosen questions they will give the same answer to both questions at least 1/3 of the time.

In real EPRB experiments (e.g. [1]) the results violate Bell's inequality. This shows that *no* deterministic and strongly local theory can explain the behavior of the photons (cross-hatched shading in Fig. 1). Taken together, the EPR and Bell analyses of the experimental data show that strong locality must be false. If we accept the principle of common cause (Sec. III) this means that some causal influences travel faster than light.

The rest of this paper explores the EPR and Bell analyses in as much depth as is possible without mathematical formalism. In Sec. III we lay out in more detail the meaning of the key postulates of strong locality and determinism. In Sec. IV we give an intuitive non-mathematical explanation of the Bell inequality and the resultant refutation of strong locality. Sec. V applies these concepts to the real experimental setup involving photon spin measurements.

This paper focuses on strong locality because it is clear, intuitively plausible, and can be cleanly defined as a factorization condition (Eq. (1)). However, other analyses of the EPRB experiment (e.g., [8, 9]) do not use this definition, and hence come to different-sounding conclusions about what EPRB means for "locality". In Sec. VI we therefore explore other principles that are related to locality, such as "information cannot be transmitted faster than light" or "there is no preferred inertial reference frame" (the Principle of Relativity), and discuss how some form of locality might survive even when strong locality is violated. Sec. VII gives a summary of our discussions.

III. LOCALITY AND DETERMINISM: DEFINITIONS AND ASSUMPTIONS

The principles that play a central role in EPRB experiments are:

1. **Determinism:** The result of any measurement on a system is pre-determined by how the system was set up originally, taking into account any subsequent influences on it. Any apparent randomness just reflects our ignorance, there is no essentially random component to the outcome [10]. In a deterministic theory, even for a measurement that was not actually performed there is a fact of the matter about what result it would have yielded ("counterfactual definiteness").

2. **Strong Locality:** Once we take into account everything in its past light cone, the probability of an event is not affected by additional information about things that happened outside its past light cone (Fig. 2) [2]. This is sometimes called "factorizability" because it leads to a factorization of the probability function for space-like-separated events (Eq. (1)). As we will explain below, using a reasonable-seeming conception of "cause" it is equivalent to saying that causal influences cannot travel faster than light, so the causal influences that affect an event must be in its past light cone.

We now explain in more detail our background assumptions and the meaning of determinism and strong locality. Readers interested in getting straight to the EPR and Bell analyses can skip the rest of this section.

Background Assumptions
In our discussion we will make the following background

assumptions. For a more fine-grained formulation see Ref. [5].

1. "Macro-realism": each measurement has a unique outcome.

2. "Random choices": each experimenter's choice of what to measure is random, i.e., uncorrelated with the state of the particles being measured and choices made by the other experimenter.

These allow us to conclude from EPRB experiments that strong locality is violated. To make a connection between strong locality and causal influences, one needs

3. Reichenbach's principle of common cause [11]: correlations can be explained in terms of causes. if two phenomena show a correlation, either one causes the other or they have a common cause. If C is the common cause of A and B then conditioning on C factorizes the joint probability: $p(A,B) = p(A|C)\,p(B|C)$.

These assumptions seem reasonable but not incontrovertible [3, 5, 12–14]. Proponents of many-worlds-type scenarios would deny macro-realism. A superdeterminist or a believer in retrocausality would not allow us to assume that the experimenters choices can be treated as random. Operationalists deny Reichenbach's principle. We will comment further on these viewpoints in Sec. VII.

Determinism

Determinism states that the outcome of a measurement is predetermined by the state of the system at earlier times, taking into account any external influences on it. In the context of EPRB experiments, as we will see in Secs. IV and V, determinism means that the outcome of doing a measurement on a particle can be reliably "preprogrammed" by physical processes that set the initial states of two particle before they are moved apart from each other.

Determinism is intimately bound up with our understanding of *uncertainty.* One can distinguish two ways in which we may be uncertain about the outcome of a measurement:

1. Uncertainty arising from our ignorance. The outcome of the measurement could be predicted given accurate knowledge of the initial state of the object and the laws governing its evolution, but we don't have sufficiently accurate information about these things to make an exact prediction.

2. Fundamental uncertainty: the outcome of the measurement has an essentially random component, either in the evolution of the system or its effect on the measuring device. In a sense the system gets to "decide on its own" how to behave.

In ordinary life, and in science up until the advent of quantum mechanics, all the uncertainty that we encounter is presumed to be of the first kind, uncertainty

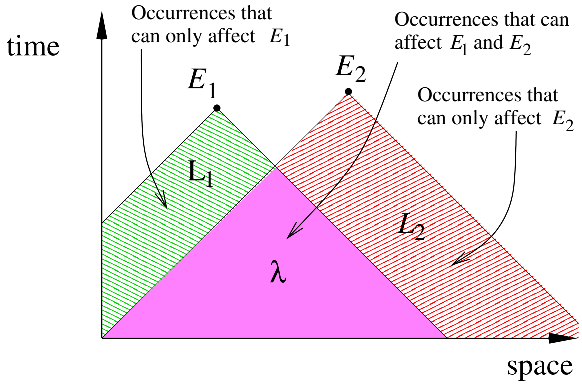

FIG. 2: Strong locality states that correlations between the results of two spacelike-separated measurements E_1 and E_2 can only arise from events in their shared past light cone.

arising from ignorance. We can't predict the weather very accurately, but the more we learn about the state of Earth's atmosphere and oceans and the laws they obey, the better our predictions become. Determinism says that *all* uncertainty is of the first kind, the kind that arises only from our ignorance. Determinism is a sort of scientific optimism: if we knew enough about the state of the universe we could predict the outcome of any measurement.

Quantum mechanics introduced the idea that there might be uncertainty of the second type, that nature might be fundamentally non-deterministic. The EPR analysis shows that if strong locality is valid then this sort of uncertainty is in conflict with the outcome of EPRB experiments.

Strong Locality

The application of strong locality to the EPRB experiment is sketched in Fig. 2. Formally, it says that any correlation between two spacelike separated events E_1 and E_2 can only arise from each of them being correlated with events λ in their shared past light cone. Once we take into account those shared influences the joint probability distribution of E_1 and E_2 factorizes [3, 5, 10]:

$$p(E_1, E_2 \,|\, L_1, L_2, \lambda) = p(E_1 \,|\, L_1, \lambda) \times p(E_2 \,|\, L_2, \lambda) \quad (1)$$

where

E_1 is the outcome of the measurement on photon 1
E_2 is the outcome of the measurement on photon 2
L_1 is events in the past light cone of E_1 but not E_2
L_2 is events in the past light cone of E_2 but not E_1
λ is everything in both light cones, or any other state of affairs that can affect both E_1 and E_2

(Given strong locality, determinism is the statement that $p(E_1 \,|\, L_1, \lambda)$ is zero or one, and similarly for $p(E_2 \,|\, L_2, \lambda)$.)

As we will see in Secs. IV and V, for the EPRB experiments to falsify strong locality each experimenter must decide "at the last minute" what experiment to do on her photon. Thus the decision of what measurement to perform on photon 1 occurs in L_1, so if strong locality is true then that decision should not affect the measurement on photon 2, and vice versa. The assumption of random choices (Sec. III) is crucial here; we assume that the choices made by the experimenters are not influenced by the events λ that determine the state of the photons, hence the random choices assumption is often called "lambda-independence" [12].

If we accept Reichenbach's principle of common cause then the violation of strong locality means that there must be some faster-than-light causal influence that allows the measurement of one photon to affect the measurement of the other [5, 15, 16].

IV. EPR AND BELL WITH HUMANS

In order to make the EPR and Bell analyses of the EPRB data as comprehensible as possible we now explain them using an analogy where instead of experimenting on photons we are questioning people. For related approaches see, e.g., Sec 4.1.3 of Ref. [17], or Ref. [18].

A. Testing twins for superluminal telepathy

Imagine that someone has told us that twins have special powers, including the ability to communicate with each other using telepathic influences that are "superluminal" (faster than light). We decide to test this by collecting many pairs of twins, separating each pair, and asking each twin one question to see if their answers agree.

To make things simple we will only have three possible questions, and they will be Yes/No questions. We will tell the twins in advance what the questions are.

The procedure is as follows.

1. A new pair of twins is brought in and told what the three possible questions are.
2. The twins travel far apart in space to separate questioning locations.
3. At each location there is a questioner who selects one of the three questions at random, and poses that question to the twin in front of her.
4. **Spacelike separation**: When the question is chosen and asked at one location, there is not enough time for any influence travelling at the speed of light to get from there to the other location in time to affect either what question is chosen there, or the answer given.

B. EPR analysis of the data for the twins

Now, suppose we perform this experiment and we find **same-question agreement**: whenever a pair of spacelike-separated twins both happen to get asked the same question, their answers always agree. How could they do this? There are two possible explanations,

1. Each pair of twins uses superluminal telepathic communication to make sure both twins give the same answer.
2. Each pair of twins follows a plan. Before they were separated they agreed in advance what their answers to the three questions would be.

The same-question agreement that we observe does not prove that twins can communicate telepathically faster than light. If we believe that strong locality is a valid principle, then we can resort to the other explanation, that each pair of twins is following a plan. The crucial point is that this requires determinism. If there were any indeterministic evolution while the twins were spacelike separated, strong locality requires that the random component of one twin's evolution would have to be uncorrelated with the other twin's evolution. Such uncorrelated indeterminism would cause their recollections of the plan to diverge, and they would not always show same-question agreement. This inference corresponds to the EPR analysis of the EPRB experiment: strong locality (the twins cannot exchange information faster than light), when combined with same-question agreement, implies determinism (each pair of twins follows a predefined plan).

The idea that twins use a deterministically-evolving internal "memory" in order to follow a plan does not seem so remarkable, but for photons this is a striking claim, because the quantum mechanical picture of a photon does not allow for any internal state that determines the outcome of measurements on a photon. The conclusion of the EPR analysis (vertical shading in Fig. 1) is that if nature obeys strong locality then only a deterministic theory can account for the agreement behavior seen in EPRB experiments.

C. Bell inequality for the twins

In the thought experiment as described up to this point we only looked at the recorded answers in cases where each twin in a given pair was asked the same question. There are also recorded data on what happens when the two questioners happen to choose different questions. Bell noticed that this data can be used as a cross-check on our strong-locality-saving idea that the twins are following a pre-agreed plan that determines that their answers will always agree. The cross-check takes the form of an

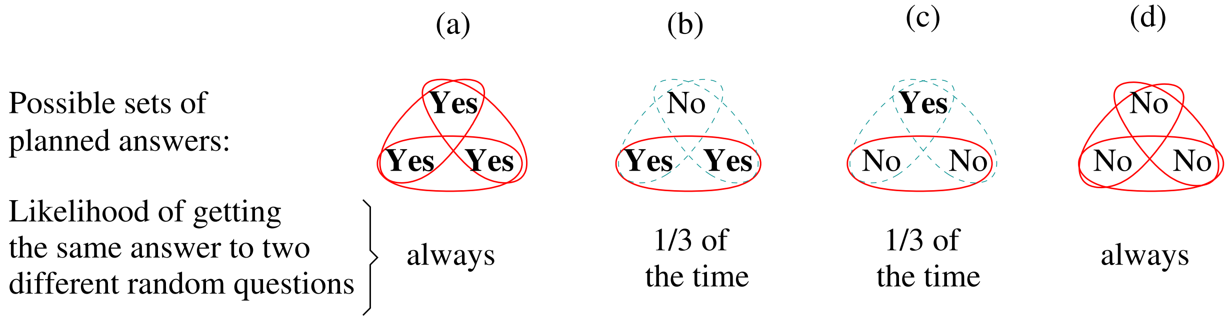

FIG. 3: The essence of the Bell inequality (Eq. (2)). In formulating a plan for how to give Yes/No answers to three questions, there are four types of plan. No matter what plan one follows, the answers to two different randomly chosen questions will be the same *at least 1/3 of the time.*

inequality:

Bell inequality for twins:

If a pair of twins is following a plan then, when each twin is asked a *different* randomly chosen question, their answers will be the same, on average, at least 1/3 of the time. (2)

Fig. 3 illustrates why (2) is true. For each pair of twins, there are four general types of pre-agreed plan they could adopt when they are arranging how they will both give the same answer to each of the three possible questions.
(a) a plan in which all three answers are Yes;
(b) a plan in which there are two Yes and one No;
(c) a plan in which there are two No and one Yes;
(d) a plan in which all three answers are No.
If, as strong locality and same-question agreement imply, both twins in a given pair follow a shared predefined plan, then when the random questioning leads to each of them being asked a *different* question from the set of three possible questions, how often will their answers happen to be the same (both Yes or both No)? If the plan is of type (a) or (d), both answers will always be the same. If the plan is of type (b) or (c), both answers will be the same 1/3 of the time. We conclude that no matter what type of plan each pair of twins may follow, the mere fact that they are following a plan implies that, when each of them is asked a different randomly chosen question, they will both give the same answer (which might be Yes or No) at least 1/3 of the time (Eq. 2). It is important to appreciate that one needs data from many pairs of twins to see this effect, and that the inequality holds even if each pair of twins freely chooses any plan they like.

This, then, is how the Bell analysis applies to the data for the twins: strong locality (no way for the twins or questioners to influence each other when the questioning is happening) and determinism (each pair of twins follows a plan) implies a Bell inequality (2).

D. What if the twins violate the Bell inequality?

In real experiments, when performing the analogous experiment on photons, the Bell inequality is violated, showing that no strongly local and deterministic theory can explain the data (cross-hatched shading in Fig. 1).

Let us imagine the same thing happening in our analogy. Suppose that when we analyze our results for a large sample of twins, we find that in cases where each twin was asked a different question, their answers are the same only 1/4 of the time; 3/4 of the time one twin gives a Yes and the other a No. This result violates the Bell inequality (2), and tells us that a good fraction of the population of twins was *not* following any predefined plan when they answered the questions. How do we interpret this result?

Our goal was to see if there was any evidence that the twins were communicating with each other using telepathic influences that travel faster than light. The fact that the twins always agree when they are both asked the same question, even when they are being interrogated at spacelike separated locations, could be explained away by assuming they were following a prearranged plan. But if their pattern of answers to different questions violates the Bell inequality then this shows that they can't be following a prearranged plan. When one twin answers the question posed to him, he needs to know what question his twin is being asked, because if his twin is being asked a different question, at least some of the time one of them will have to deviate from any pre-arranged plan, changing his answer in such a way that it differs from the answer that his brother is giving, and thereby allowing their responses to violate the Bell inequality. Unless we are willing to discard one of the background assumptions listed in Sec. III, we are forced to accept that some sort of superluminal influence connects the twins.

V. EPR AND BELL WITH PHOTONS

The testing of twins for telepathic abilities, as described in section IV, is an exact analogy to the EPRB experiment, which is a modification, suggested by Bohm

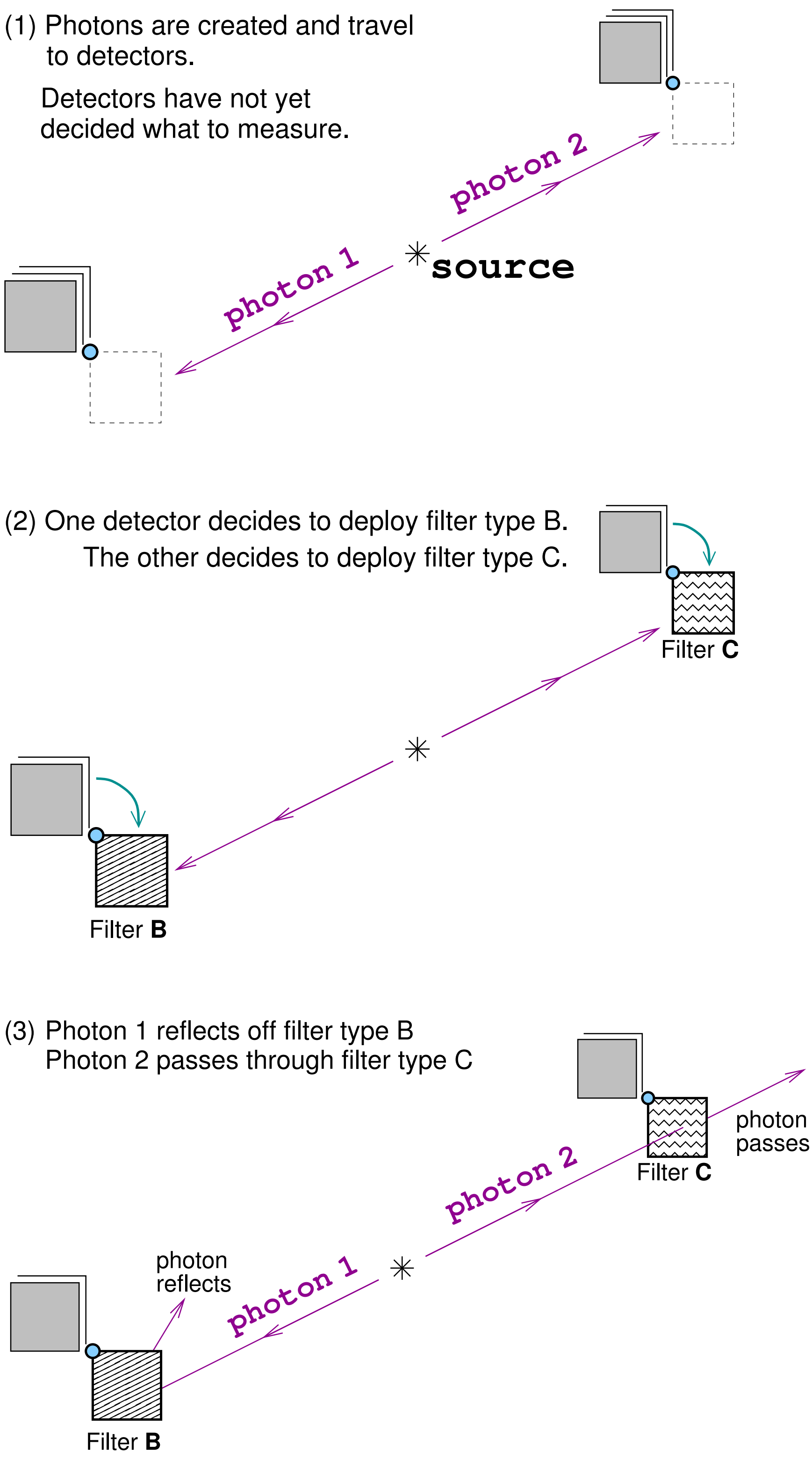

FIG. 4: One trial in the EPRB measurement of polarization for two photons. The final result in this trial is that photon 1 encountered a filter of type B and reflected off it, while photon 2 encountered a filter of type C and passed through it. According to the Bell inequality (Eq. (3)) this sort of result, where the two photons do different things when encountering different filters, should happen no more than 2/3 of the time.

[19], of the original EPR experiment. In the EPRB experiment (see Fig. 4) there is a source that creates pairs of photons, analogous to twins. The photons travel out from the source in opposite directions. When they are far from each other, each photon encounters a measuring machine that can do three possible measurements. The machine contains three types of filter, call them A,B, and C, and when the photon arrives the machine flips one of the three types of filter into the path of the photon. The photon has two possible responses to the filter: it either goes through the filter ("+") or reflects off it ("−"). This is actually a measurement of the polarization of the photon: each filter consists of polaroid with a different orientation of its axis of polarization. If determinism is true then each photon has a deterministically evolving inherent polarization state that determines how it will interact with each filter.

If both machines deploy the same filter then we see "same-axis agreement": either both photons pass through or they both reflect off.

As with the twins, we can immediately see two ways to explain this consistent agreement.

1. *Influence*: when one photon reaches its machine and the machine decides what filter to flip up in front of it and the photon responds to that filter, some information is superluminally transmitted to the other photon so that if the other photon gets the same filter, it will behave the same way.

2. *Determinism*: when the photons are created, each is formed in a state (its "polarization state") that determines how it will respond to any possible filter it might encounter. The source puts both photons into the same state, and those states evolve deterministically, ensuring that the photons always behave the same way when they encounter the same type of filter.

The EPR analysis (vertical shading in Fig. 1) concludes that in any strongly local theory, since there are no faster-than-light correlation-creating influences, agreement in same filter (same axis) measurements must arise from the photons having a deterministically evolving internal state that pre-determines their response to the filters that they encounter. If, as EPR did, one takes strong locality to be valid, then the observed same-axis agreement shows that the photons are in a state that determines their behavior, which is in contradiction with the quantum mechanical picture where their state does not determine the outcome of measurements performed on them.

However, just as for the twins, there is a Bell analysis (cross-hatched shading in Fig. 1) which shows that EPR's picture, of physical objects having deterministically evolving states and strongly local interactions, can be experimentally tested. For this we look at the data for the cases when the two measuring machines deploy *different* filters in front of the two photons. Following the logic used in Sec. IV, we conclude that if both photons are in the same polarization state, and there is no correlation-creating influence between their spacelike-separated measurements, then, on the occasions that the detectors deploy *different* filters, then photons 1 and 2 should show the same behavior (both bouncing off or both passing through) at least 1/3 of the time:

Bell inequality:

$$\text{prob}\begin{pmatrix}\text{when photon 1 and photon 2} \\ \text{encounter different filters,} \\ \text{they show the same behavior}\end{pmatrix} \geqslant 1/3 \tag{3}$$

In Appendix A we show how Eq. (3) is a form of Bell's original inequality.

When polarizations of pairs of spin-singlet photons are measured in real-world experiments, it is found that they do show agreement in same-axis measurements, but when we perform different-axis measurements the two photons only show the same behavior 1/4 of the time; 3/4 of the time they show different behavior: one bounces off its filter and the other passes through. This violates the Bell inequality. Such violation has now been seen in many experiments, e.g. [1, 20, 21].

We conclude that strong locality is violated by spin-singlet photon pairs. Either you need a strong-locality-violating influence to make the same-axis agreement happen, or, if you try to save strong locality by assuming that each photon is in a state (the same state for both of them) that pre-determines the outcome of measurements on it, then you need a strong-locality-violating influence to obtain the observed violation of Bell's inequality for different-axis measurements. Either way, a violation of strong locality is required to account for all the relevant experimental observations.

VI. CONSEQUENCES FOR LOCALITY

The EPRB experiment, in combination with some assumptions that we have outlined in Sec. III, tells us that nature does not obey the principle of strong locality. If we accept Reichenbach's principle of common cause we would say that there are causal influences that travel faster then light. But this cannot be the end of the story:
• What about Einstein's theory of relativity? Are EPRB results compatible with the Principle of Relativity?
• If so, is there some "medium-strength" locality principle, implied by Relativity but weaker then Strong Locality, that is compatible with EPRB experiments?
• What about determinism? Do the EPR and Bell analyses leave open the possibility of deterministic theories?

We will now explain why it is believed that EPRB experiments do not violate the Principle of Relativity, and suggest that "signal locality" is a useful medium-strength locality principle, since it distills the requirements of relativity and chronology protection (the absence of causal paradoxes [22]). We will acknowledge that signal locality contains concepts such as "control" that are not usually present in physical principles, and argue that, although signal locality is compatible with determinism, nature must have some inherent elusiveness, perhaps indeterminism or perhaps some form of uncontrollability, in order for the EPRB results to be consistent with signal locality and hence with Relativity and chronology protection.

A. EPR and the Principle of Relativity

To quote Bell himself, *"one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics—the real problems of quantum mechanics—they must be talking about Lorentz invariance"* [23]. In this quote, "Lorentz invariance" is just the Principle of Relativity, which states that the laws of physics are the same in all inertial reference frames, so the laws of physics are invariant under the Lorentz transformations that relate different reference frames to each other.

So, is the faster-than-light connection between distant photons that we see in EPRB experiments compatible with the Principle of Relativity? There is evidence that they are compatible, but not in the straightforward way that one might assume.

Naively one might say that of course the EPRB results are consistent with the Principle of Relativity, because they agree with the predictions of quantum mechanics, and there is a relativistic, Lorentz-invariant, formulation of quantum mechanics, namely quantum field theory. It is true that most presentations of quantum field theory seem Lorentz invariant because they focus on expectation values and do not discuss the measurement postulate (instantaneous wavefunction collapse induced by the measurement process). But textbook quantum mechanics, including quantum field theory, needs the measurement postulate to explain how unique experimental results arise from measurements (the "macro-realism" assumption of Sec. III, discussed further in Sec. VII). There is no Lorentz-invariant version of measurement-induced wavefunction collapse that is compatible with the EPRB results [3]. However, this does not rule out the possibility that there may be other Lorentz-invariant theories that can explain the EPRB results. In fact, in 2006 an example was proposed: a version of quantum mechanics where the wavefunction occasionally collapses spontaneously in a Lorentz-invariant way [24]. Whether or not this theory is a correct description of nature, it seems to provide an existence proof that EPRB results are compatible with the Principle of Relativity.

B. Different forms of locality

If the Principle of Relativity is compatible with EPRB experiments but strong locality is not, then strong locality does not follow from the Principle of Relativity. However, there is another locality principle, *signal locality*, that does plausibly follow from relativity combined with chronology protection (no causal paradoxes). To set the context for our discussion of signal locality, here is a quick survey of various requirements that can be thought of as expressing ideas of locality, along with a summary of how compatible they are with EPRB experiment results:

1) *Strong locality* (or *local causality*): after taking into account everything in its past light cone, the probability of an event is not affected by additional information about things that happened outside its past light cone.
   As we have seen, this is disproven by EPRB experiments.
2) *Information* must be transmitted no faster than light.
   This is also disproven by EPRB experiments, since the result of the measurement on one photon contains information about the measurement performed on the other that did not come from the backward light cone.
3) *Signal locality*: signals can travel no faster than light.
   This is compatible with EPRB experiments, but at a price, as we will describe below.
4) *Energy or other conserved quantities* must travel no faster than light.
   This is compatible with EPRB experiments, since there is no evidence that any physical substance travels from one photon's measurement site to the other's.
5) *The Principle of Relativity*: The laws of physics are the same for any observer who is not accelerating (any "inertial frame of reference").
   As discussed above, this is compatible with EPRB experiments.

C. EPR and Signal locality

In discussing signals, the essential point is that signalling is more than the transfer of information. Sending a signal means having a *controllable* means of transferring information. Control, however, is based on high-level concepts such as agency and free will, and such concepts are not usually invoked in fundamental physical principles. Bell complained that signal locality "rests on concepts which are desperately vague, or vaguely applicable. The assertion that 'we cannot signal faster than light' immediately provokes the question: Who do we think *we* are?" [25].

In view of this concern, we will proceed by treating signal locality as a property (of theories) that we hope will be implemented by some more fundamental feature of the theory. Theories with the property of signal locality are attractive because they can obey the Principle of Relativity (no preferred inertial frame of reference) while maintaining chronology protection, i.e. avoidance of causal paradoxes. There is danger of a causal paradox if someone can send a signal to themselves in the past, since the person could, after receiving the signal, decide

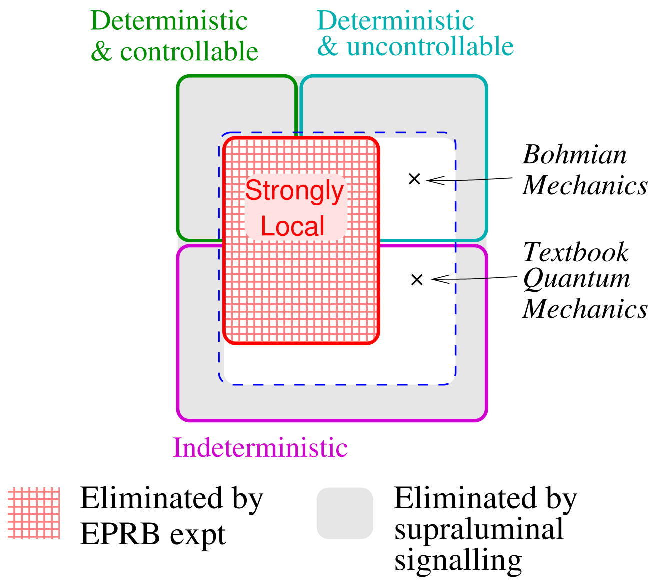

FIG. 5: Augmented version of Fig. 1, showing the set of theories obeying signal locality (enclosed by the dashed (blue) line), and dividing deterministic theories into those where the hidden variables can be controlled and those where they cannot. The grey shaded region is the set of theories where superluminal signalling is possible. Theories in the white area are consistent with EPRB experiments and have the right kind of indeterminism/uncontrollability to be consistent with signal locality. For details see Sec. VI C.

(assuming free will) not to send it. (For a discussion not assuming free will see Ref. [26].) To make sure that this cannot happen, we have to ensure that the sender of controllable information (signals) is always in the past of the receiver. In a theory that obeys the Principle of Relativity, this means that signals must go slower than light, since only then will all reference frames agree on the time ordering of the sender and receiver.

We can now see that EPRB experiments, which show that information can be transferred faster than light, also require some accompanying element of uncontrollability in nature in order to make sure that this does not translate to a violation of signal locality. We can imagine two ways to preserve signal locality in the face of the results of EPRB-type experiments.

(a) *Nature is indeterministic.*
The measurement outcomes are uncontrollable because nature is fundamentally indeterministic. In an indeterministic theory there is a random component to the evolution and/or measurement of the photons, so we have to rely on some sort of faster-than light influence to produce the consistent agreement in the results of space-like separated same-axis measurements and also to produce the different-axis correlations that violate the Bell inequality. But this does not allow superluminal signalling because the measurement results are not determined by anything: they are inherently random and therefore uncontrollable. In this case signal locality follows from the requirement of "parameter independence" (see Appendix B). Textbook quantum mechanics is an example of an indeterministic parameter-independent theory. The instantaneous collapse of the wavefunction is the faster-than-light influence that ensures same-axis agreement.

(b) *Nature is deterministic but uncontrollable.*
The measurement outcomes are uncontrollable because, even though they are determined by the states of the objects in question ("hidden variables"), those states are themselves sufficiently uncontrollable, because of some physical law, that they cannot be used to send signals. In such a scenario the violation of Bell's inequality in the different-axis measurements arises from a faster-than-light influence that allows one measurement to affect the hidden variables of the object being measured at the other, space-like separated, location, but the experimenter at one location can never control the hidden variable states well enough to be able to control the measurement results at the other end, and thereby send a message. Bohmian Mechanics is an example of a theory that follows this pattern [3, 12].

In Appendix B we describe Jarrett's more formal way of exposing this dichotomy, by analyzing strong locality into two weaker conditions [10]. Violation of one of them ("Remote Outcome Independence") corresponds to possibility (a) above; violation of the other ("Parameter Independence" or "Remote Detector Independence") corresponds to possibility (b).

In Fig. 5 we show an augmented version of the simple theory-space diagram (Fig. 1), including the set of theories that obey signal locality (enclosed by the dashed (blue) line). If signal locality is valid then physics is restricted to the unshaded (white) area of allowed theories. These all violate strong locality, as required by the EPRB experimental results, but in a way that avoids superluminal signalling as described above.

VII. SUMMARY

The EPRB experiment uses spin-singlet photon pairs to test the degree to which the laws of nature obey some sort of principle of locality. If we accept some background assumptions (Sec. III) then the EPRB experimental results bring us to the following conclusions.

- The observed behavior violates the principle of strong locality (local causality) which states that no correlation-creating influence can travel faster than light. In a nutshell, this is because in the EPR experiment either you need a faster-than-light influence to make the same-axis agreement happen, or, if you try to save strong locality by assuming that the agreement arises from the photons being in states that determine in advance that their spins will have specific values, then you need a faster-than-light influence to obtain the violation of Bell's inequality for different-axis measurements.

- The EPRB results are compatible with the Principle of Relativity (equivalence of all inertial reference frames, also called Lorentz invariance).
- In order to avoid casual paradoxes we expect nature to display the property of *signal locality* (signals cannot travel faster than light). This means there must be sufficient indeterminism or uncontrollability in nature to prevent the EPRB correlations from being used for signalling.

If we prefer to explain the EPRB results by adopting an indeterministic theory (such as textbook quantum mechanics) then, while accepting that strong locality is violated, we can ensure that the theory is signal-local by imposing a weaker locality principle like "parameter independence", although, as described in Appendix B, the concept of parameter independence also involves non-fundamental concepts of the type that made Bell object to signal locality as a fundamental principle. Treatments of EPRB that favor this approach (e.g. [8, 9]) tend to de-emphasize the violation of strong locality and frame EPRB as forcing us to choose between determinism and a weaker form of locality.

If we wish to preserve determinism then we must come up with a theory (such as Bohmian mechanics) in which there are superluminal influences between the deterministically evolving hidden variables, but we face the challenge of constructing the theory so that it preserves signal locality by enforcing essential limits on our ability to control those variables, so they cannot be used for signalling.

In Sec. III we listed the background assumptions used in our analysis. We now briefly discuss the possibility of dropping those assumptions. For a fuller discussion see Ref. [5].

Dropping the assumption of macro-realism (experiments have unique outcomes) renders our entire analysis moot. However, an anti-macro-realist must then explain how it is that experiments always appear to have unique outcomes. Anti-macro-realist versions of quantum mechanics such as the many-worlds [27] or many-minds [28] interpretations lead to questions of how probabilistic predictions emerge and the role of decoherence [12].

It is possible to deny that the choices made by the experimenters can be treated as random. For example, a superdeterminist (e.g. [14]) would suggest that some mechanism ensures that those choices are always predetermined just so as to violate the Bell inequality. This seems difficult, given that there are many ways to design a pseudo-random number generator for each experimenter to use, including ones that are sensitive to events outside their shared past lightcone λ (see Fig. 2 and Ref. [12]). Alternatively, a believer in retrocausality would suggest that the experimenters' choices could exert influence backwards in time on the state in which the particles were prepared. This calls for an explanation of why such retrocausality does not lead to violations of chronology protection [29].

Reichenbach's principle of common cause is not an essential assumption but it plays a fundamental role in science. An "operationalist" or "instrumentalist" would reject it [15, 30], admitting that EPRB experiments violate strong locality, but maintaining that not all correlations can be explained in terms of causes that factorize correlations (see Sec. III), so this is not a sign of superluminal causal influences. However, since most of science consists of the search for the causes of correlations, the operationalist then has to explain which correlations call for such causal explanations and which do not. A Quantum Information theorist would also reject Reichenbach's principle, claiming that quantum entanglement can cause correlations without causing the individual events that exhibit the correlation [31].

In conclusion, the EPRB experiment exposes some of the complexity of the concept of locality. Strong locality, which seems simple and intuitively attractive, is violated by the results while the Principle of Relativity is not. Signal locality can be preserved, but it is formulated in terms of concepts like "control" which seem out of place in a theory of physics.

Quantum mechanics uses indeterminism to avoid superluminal signalling and impressively accounts for the unusual characteristics of the EPRB correlations (they are not attenuated with distance, and they only connect specific particles that were created in entangled states) but in textbook quantum mechanics the measurement-induced collapse of the wavefunction is instantaneous over all space and therefore not Lorentz invariant, so one natural goal is to find and empirically validate a Lorentz invariant form of wavefunction collapse such as that proposed by Tumulka [24].

The EPRB results also leave open the possibility of theories that, unlike quantum mechanics, are deterministic, with signal locality ensured by limits on the controllability of the hidden variables. Bohmian mechanics is a well known proposal, and there is an ongoing search for a Lorentz-invariant version of it [32].

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[1] B. Hensen *et. al.*, *Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km*, Nature **526** (2015) 682–686, [arXiv:1508.0594].

[2] J. S. Bell, *The theory of local beables*, Epistemological Lett. **9** (1976) 11–24.

[3] T. Maudlin, *Quantum Non-Locality and Relativity: Metaphysical Intimations of Modern Physics.* Wiley, 2011.

[4] T. Norsen, *Are there really two different Bell's theorems?*, ArXiv e-prints (Mar., 2015) [arXiv:1503.0501].

[5] H. M. Wiseman and E. G. Cavalcanti, *Causarum investigatio and the two bell's theorems of john bell*, 2015. arXiv:1503.0641.

[6] A. Einstein, B. Podolsky, and N. Rosen, *Can quantum mechanical description of physical reality be considered complete?*, Phys.Rev. **47** (1935) 777–780.

[7] J. Bell, *On the Einstein-Podolsky-Rosen paradox*, Physics **1** (1964) 195.

[8] G. Blaylock, *The epr paradox, bell's inequality, and the question of locality*, American Journal of Physics **78** (2010), no. 1 111–120.

[9] L. Maccone, *A simple proof of bell's inequality*, American Journal of Physics **81** (2013), no. 11 854–859.

[10] J. Jarrett, *Bell's theorem: a guide to the implications*, in *Philosophical Consequences of Quantum Theory: Reflections on Bell's theorem* (J. T. Cushing and E. McMullin, eds.), pp. 60–79. University of Notre Dame Press, 1989.

[11] H. Reichenbach, *The Direction of Time.* Berkeley: University of Los Angeles Press, 1956.

[12] J. Berkovitz, *Action at a distance in quantum mechanics*, in *The Stanford Encyclopedia of Philosophy* (E. N. Zalta, ed.). fall 2013 ed., 2013.

[13] F. Arntzenius, *Reichenbach's common cause principle*, in *The Stanford Encyclopedia of Philosophy* (E. N. Zalta, ed.). fall 2010 ed., 2010.

[14] G. 't Hooft, *How does god play dice? (pre)determinism at the planck scale*, hep-th/0104219.

[15] B. C. Van Fraassen, *The charybdis of realism: Epistemological implications of bell's inequality*, Synthese **52** (1982), no. 1 25–38.

[16] T. Norsen, *John S. Bell's concept of local causality*, American Journal of Physics **79** (Dec., 2011) 1261–1275, [arXiv:0707.0401].

[17] J. Preskill, *Quantum Information and Computation*, Lecture notes for Physics 229 (Caltech) http://www.theory.caltech.edu/people/preskill/ph229/, 1998.

[18] J. Bricmont, *What did Bell really prove?*, Int. J. Quant. Found. (archive) http://www.ijqf.org/archives/1303, 2014.

[19] D. Bohm, *Quantum theory.* Dover, Unknown, 1st ed., 1989.

[20] D. Salart, A. Baas, J. A. W. van Houwelingen, N. Gisin, and H. Zbinden, *Spacelike Separation in a Bell Test Assuming Gravitationally Induced Collapses*, Physical Review Letters **100** (June, 2008) 220404, [arXiv:0803.2425].

[21] M. Giustina, A. Mech, S. Ramelow, B. Wittmann, J. Kofler, J. Beyer, A. Lita, B. Calkins, T. Gerrits, S. W. Nam, R. Ursin, and A. Zeilinger, *Bell violation using entangled photons without the fair-sampling assumption*, Nature (London) **497** (May, 2013) 227–230, [arXiv:1212.0533].

[22] S. W. Hawking, *The Chronology Protection Conjecture*, Phys. Rev. **D46** (1992) 603–611.

[23] S. Goldstein, *Bohmian mechanics*, in *The Stanford Encyclopedia of Philosophy* (E. N. Zalta, ed.). spring 2013 ed., 2013.

[24] R. Tumulka, *A Relativistic Version of the Ghirardi Rimini Weber Model*, Journal of Statistical Physics **125** (Nov., 2006) 821–840, [quant-ph/0406094].

[25] J. S. Bell, *La nouvelle cuisine*, in *Between Science and Technology* (A. Sarlemijn and P. Kroes, eds.), pp. 97–115, North-Holland, 1990.

[26] F. Arntzenius and T. Maudlin, *Time travel and modern physics*, in *The Stanford Encyclopedia of Philosophy* (E. N. Zalta, ed.). winter 2013 ed., 2013.

[27] H. Everett, J. Barrett, and P. Byrne, *The Everett Interpretation of Quantum Mechanics: Collected Works 1955-1980 with Commentary.* high everett. Princeton University Press, 2012.

[28] D. Albert, *Quantum Mechanics and Experience.* Harvard University Press, 2009.

[29] D. T. Pegg, *Retrocausality and quantum mechanics*, Studies in History and Philosophy of Science Part B: Studies in History and Philosophy of Modern Physics **39** (2008), no. 4 830 – 840.

[30] A. Fine, *Do correlations need to be explained?*, in *Philosophical Consequences of Quantum Theory: Reflections on Bell's theorem* (J. T. Cushing and E. McMullin, eds.), pp. 175–194. University of Notre Dame Press, 1989.

[31] L. Maccone. Personal Communication, 2015.

[32] D. Drr, S. Goldstein, T. Norsen, W. Struyve, and N. Zangh, *Can Bohmian mechanics be made relativistic?*, Proc.Roy.Soc.Lond. **A470** (2013) 20130699, [arXiv:1307.1714].

[33] W. Dickson, *Quantum chance and non-locality.* Cambridge University Press, 1998.

Appendix A: Relationship to Bell's original inequality

Here we describe how our informally derived inequality Eq. (3) follows from the mathematical inequality derived by Bell. Let us define

$$p(+-|AB) \equiv \text{probability that, \textit{given} that machines 1 and 2 have decided to deploy filters A and B respectively, photon 1 passes through and photon 2 bounces off} \quad \text{(A1)}$$

and so on. Then Bell's original inequality is

$$\begin{aligned} &p(+-|AB) + p(+-|BC) + p(+-|CA) \leqslant 1 \\ &\text{or, equivalently,} \\ &p(-+|AB) + p(-+|BC) + p(-+|CA) \leqslant 1\,. \end{aligned} \quad \text{(A2)}$$

To derive our inequality (3) from Bell's original inequality, start by rewriting (3) as

$$\begin{aligned} p_{\text{diff}} &\leqslant 2/3 \\ \text{where} \\ p_{\text{diff}} &= \mathrm{p}\begin{pmatrix} \text{when photon 1 and photon 2} \\ \text{encounter different filters,} \\ \text{they show different behavior} \end{pmatrix}, \end{aligned} \quad \text{(A3)}$$

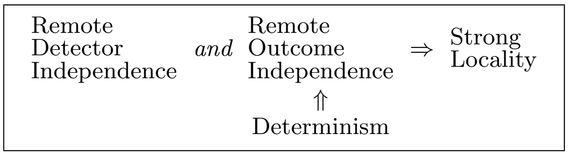

FIG. 6: Strong locality can be understood as saying that the outcome at one detector is independent of both the settings of the remote detector, and the outcome of the remote measurement [10].

then using the notation (A1) and defining $p(AB)$ to be the probability that machine 1 deploys the A filter and machine 2 deploys the B filter, and so on, we can rewrite $p_{\rm diff}$ as a sum over all filter settings $F = (AB, BC, CA, BA, CB, AC)$ where the two detectors deploy different filters:

$$p_{\rm diff} = \frac{\sum_F p(F)\Big(p(+-|F) + p(-+|F)\Big)}{\sum_F p(F)} \ . \qquad \text{(A4)}$$

In our experiment, each filter is deployed at random, so all six combinations occur with equal probability,

$$p_{\rm diff} = \frac{1}{6}\sum_F \Big(p(+-|F) + p(-+|F)\Big) \ . \qquad \text{(A5)}$$

The labeling of photons and measuring machines as "1" and "2" is arbitrary, so with no loss of generality we can treat the BA filter deployment as being AB with the numbering of the photons and machines reversed, so $p(+-|AB) = p(-+|BA)$ and so on, so (A5) can be written

$$\begin{aligned} p_{\rm diff} = \frac{1}{3}\Big(& \ p(+-|AB) + p(+-|BC) + p(+-|CA) \\ + & \ p(-+|AB) + p(-+|BC) + p(-+|CA)\Big) \ . \end{aligned} \qquad \text{(A6)}$$

Using Bell's original inequality (A2) we recover our inequality (A3).

Appendix B: Remote Detector vs. Remote Outcome independence

As we saw in Sec. VI C, there are two forms of non-controllability that would allow us to keep the desirable principle of signal locality in the face of the EPRB experiment's results. Jarrett [10] pointed out one way to approach this. As illustrated in Fig. 6, strong locality can be written as a combination of two requirements on the outcome of measurements at a given detector.

1. *Parameter independence*, which could be more informatively described as *Remote Detector independence*: the outcome of the measurement of one photon is not affected by the detector setting for the other photon, but may be affected by the measurement outcome of the other photon. As we will describe below, this form of locality is compatible with EPR experiments and signal locality, but requires us to give up determinism.

2. *Remote Outcome independence*: the outcome of the measurement of one photon is not affected by the outcome of the remote measurement, but may be affected by the remote detector setting. This allows us to keep determinism, but requires some limits on our ability to control the hidden variables whose states determine measurement outcomes.

We can then understand EPR experiments as allowing us keep no more than one of these requirements.

The possibilities are:

(a) Parameter independence is preserved, but Remote Outcome independence is violated. This corresponds to possibility (a) in Sec. VI C. We have to give up determinism because Remote Outcome independence can be shown to follow from determinism (Fig. 6). Since the results of measurements are independent of the controllable aspects of the remote experiment (its detector settings) we preserve signal locality. The violation of strong locality is achieved via a superluminal influence that allows the (indeterministic and therefore uncontrollable) *outcome* of one measurement to influence the other. Parameter independence is a nice guarantee of signal locality, but, as Bell pointed out (see the quote at the start of Sec. VI C), it is unclear what is the fundamental basis for this partition of phenomena into "parameters" and "outcomes". Quantum Mechanics is an example of a theory that preserves parameter independence while violating Remote Outcome independence.

(b) Remote Outcome independence is preserved, but Parameter independence is violated. This corresponds to (b) in Sec. VI C. Allowing measurement results to be influenced by the settings of the remote detector allows us to account for the EPRB results while keeping determinism, so there are hidden variables whose state determines the outcome of measurements, but to keep signal locality there must be essential limits on our ability to control the state of the hidden variables, and it is not clear what general physical principle would ensure this. Bohmian Mechanics is an example of a theory that, *pace* Dickson [33], is usually said to violate Parameter independence while preserving Remote Outcome independence [3].