QUANTUM MECHANICS I (523)
PROBLEM SET 11 (hand in December 11)

(41) Diagonalize the operator $s^2$ in the basis $\{|m_1 m_2\rangle\}$ of uncoupled spin-1/2 states, where $s = s_1 + s_2$. Verify that your results are the same as those derived in class by using lowering operators. Determine the possible eigenvalues of the operator $s_1 \cdot s_2$.

(42) Define the radial operators

$$ p = \sqrt{\mathbf{p} \cdot \mathbf{p}} $$

and

$$ r_p = \frac{1}{2} (\hat{\mathbf{p}} \cdot \mathbf{r} + \mathbf{r} \cdot \hat{\mathbf{p}}). $$

a) Show that

$$ [p, r_p] = -i\hbar. $$

b) Verify that

$$ \ell^2 = p^2(r^2 - r_p^2). $$

c) Show that in the momentum representation

$$ r_p = i\hbar \left( \frac{\partial}{\partial p} + \frac{1}{p} \right). $$

(43) Add two angular momenta with $j_1 = 1$ and $j_2 = 1$ to obtain states with total angular momentum $j = 2, 1, \text{and } 0$. Use either the ladder operator method or the recursion relation to express all (nine) eigenkets $\{|jm\rangle\}$ in terms of the uncoupled kets $\{|j_1 m_1; j_2 m_2\rangle\}$. Make sure that the coupled states are properly normalized.

(44) Consider a particle with orbital angular momentum $\ell = 0$ in the central potential

$$ V(r) = \frac{-V_0}{\exp \{\kappa r\} - 1} $$
called Hulthen's potential. Find the lowest energy eigenvalue using the operator method discussed in class for the three-dimensional oscillator and the hydrogen-like hamiltonian. Try

$$G_{\ell=0}^+ \approx p_r + ib_0 + \frac{ic_0}{\exp \{\kappa r\} - 1}$$

with $b_0$ and $c_0$ constants.