QUANTUM MECHANICS II (524)

PROBLEM SET 1 (hand in January 22)

(1) Consider a spherical tensor of rank 1 (that is, a vector)

\[ V^{(1)}_{\pm 1} = \mp \frac{1}{\sqrt{2}} (V_x \pm iV_y), \quad V^{(1)}_0 = V_z. \]

Using the expressions (see notes last Semester) of the matrix elements
\[ r^{(1)}_{m,m'}(\beta) \]
representing a rotation of a spin 1 system by an angle \( \beta \) about the \( y \)-axis, evaluate

\[ \sum_{m'} r^{(1)}_{m,m'}(\beta) V^{(1)}_{m'} \]

and demonstrate that your results coincide with what you expect from the transformation properties of \( V_{x,y,z} \) under rotations about the \( y \)-axis.

(2) Construct a spherical tensor of rank 1 out of two different vectors
\( \mathbf{F} = (F_x, F_y, F_z) \) and \( \mathbf{G} = (G_x, G_y, G_z) \). Write the components of the resulting tensor \( T^{(1)}_q \) in terms of the \( x, y, \) and \( z \)-components of \( \mathbf{F} \) and \( \mathbf{G} \).

(3) Consider the following matrix elements

\[ \langle n' \ell' m' | \mp \frac{1}{\sqrt{2}} (x \pm iy) | n \ell m \rangle \]

and

\[ \langle n' \ell' m' | z | n \ell m \rangle. \]
Relate these matrix elements as much as possible by using only the Wigner-Eckart theorem. State under what conditions these matrix elements are nonvanishing.

(4) This problem involves spherical tensors of rank 2.

a) Write $xy$, $xz$, and $(x^2 - y^2)$ as components of a spherical tensor of rank 2.

b) The expectation value

$$Q \equiv e \langle \alpha jm = j \mid (3z^2 - r^2) \mid \alpha jm = j \rangle$$

is known as the quadrupole moment. Determine

$$e \langle \alpha jm' \mid (x^2 - y^2) \mid \alpha jm = j \rangle ,$$

(where $m' = j, j - 1, \ldots$) in terms of $Q$ and appropriate Clebsch-Gordan coefficients.