QUANTUM MECHANICS II (524)

PROBLEM SET 2 (hand in January 29)

(5) Consider a system made up of two spin 1/2 particles in a spin-singlet state (meaning the total spin \( S = 0 \)). Observer A measures spin components of particle 1 while B does the same for particle 2.

a) Determine the probability for A to obtain the spin up in the 
z-direction when B makes no measurement. Same for the \( x \)-direction.

b) Observer B obtains the spin of particle 2 to be up in the \( z \)-direction with certainty. What can be concluded about the outcome of observer A’s measurement if (i) A measures \( s_{1z} \), and (ii) A measures \( s_{1x} \)? Justify your answer.

(6) Let \( \mathcal{T}_d \) denote the translation operator with displacement vector \( d \); \( R(\hat{n}; \phi) \) the rotation operator about the axis characterized by \( \hat{n} \) and by an angle \( \phi \); and \( \Pi \) the parity operator. Which, if any, of the following pairs commute and why?

a) \( \mathcal{T}_d \) and \( \mathcal{T}_{d'} \) (\( d \) and \( d' \) are in different directions).

b) \( R(\hat{n}; \phi) \) and \( R(\hat{n}'; \phi') \) (\( \hat{n} \) and \( \hat{n}' \) are in different directions).

c) \( \mathcal{T}_d \) and \( \Pi \).
d) \( R(\hat{n}; \phi) \) and \( \Pi \).

(7) Consider a spin 1/2 particle bound in a spherically symmetric potential well (like for hydrogen or the 3-D oscillator). Define spin-angular functions in two-component form as follows

\[
Y^{j=\ell \pm 1/2, m}_{\ell} = \pm \sqrt{\frac{\ell \pm m + 1/2}{2\ell + 1}} Y_{\ell,m-1/2}(\theta, \phi) \chi_+ + \sqrt{\frac{\ell \pm m + 1/2}{2\ell + 1}} Y_{\ell,m+1/2}(\theta, \phi) \chi_-.
\]

a) Write out (in as simple a form as possible) the spin-angular function \( Y_{\ell=0}^{j=1/2, m=1/2} \).

b) Express \((\sigma \cdot \mathbf{r}) Y_{\ell=0}^{j=1/2, m=1/2}\) in terms of other \( Y_{\ell}^{j,m} \).

c) Show that your result in b) is understandable by considering the transformation properties of the operator \( \mathbf{s} \cdot \mathbf{r} \) under rotations and parity.

(8) A quantum state represented by \( |\Psi\rangle \) is known to be a simultaneous eigenket of the Hermitian operators \( A \) and \( B \) which anticommute,

\[
AB + BA = 0.
\]

What can you say about the eigenvalues of \( A \) and \( B \) for the state \( |\Psi\rangle \)? Illustrate your point using the parity operator \( \Pi \) and the momentum operator \( \mathbf{p} \).