(17) A $p$-orbital electron characterized by $|n, \ell = 1, m_\ell = \pm 1, 0\rangle$ (ignore spin) is subjected to a potential

$$V = \lambda(x^2 - y^2),$$

with $\lambda$ constant.

a) Determine the correct zero-order energy eigenstates that diagonalize the perturbation. Don’t evaluate the energy shifts in detail but show that the original three-fold degeneracy is now completely removed.

b) Since $V$ is invariant under time-reversal and there is no longer any degeneracy, we expect each of the energy eigenstates obtained in a) to go into itself under time reversal (except for a phase perhaps). Check this point explicitly.

(18) A system with three unperturbed states has a hamiltonian that can be represented in that basis by

$$
\begin{pmatrix}
E_1 & 0 & a \\
0 & E_1 & b \\
a^* & b^* & E_2
\end{pmatrix},
$$
with $E_2 > E_1$. Consider the constants $a$ and $b$ to be of the same size but small compared to $E_2 - E_1$.

a) Use second-order nondegenerate perturbation theory to calculate the perturbed eigenvalues. Is this procedure correct? Comment.

b) Solve the problem exactly.

c) Now use second-order degenerate perturbation theory and compare the three results obtained.

(19) Compute the Stark effect for the $2s_{1/2}$ and $2p_{1/2}$ levels of hydrogen for a field $|E|$ sufficiently weak so that $e|E|a_0$ is small compared to the fine structure (spin-orbit splitting) but take the Lamb shift $\delta$ into account (so ignore the $2p_{3/2}$ in the calculation). Show that for $|e||E|a_0 \ll \delta$ the energy shifts are quadratic in the field strength, whereas for $|e||E|a_0 \gg \delta$ they are linear in it. The radial integral that you will require is given by $\langle 2s \mid r \mid 2p \rangle = 3\sqrt{3}a_0$. Discuss the consequences (if any) of time-reversal for this problem.

(20) Work out the quadratic Zeeman effect for the ground state of the (simple) hydrogen atom due to the usually neglected $A^2$ term in the hamiltonian. Write the energy shift as $\Delta = -1/2\chi B^2$ and obtain an expression for the diamagnetic susceptibility $\chi$. 